

Rotor dynamic response under parametric excitation: Application to the case of rolling bearings

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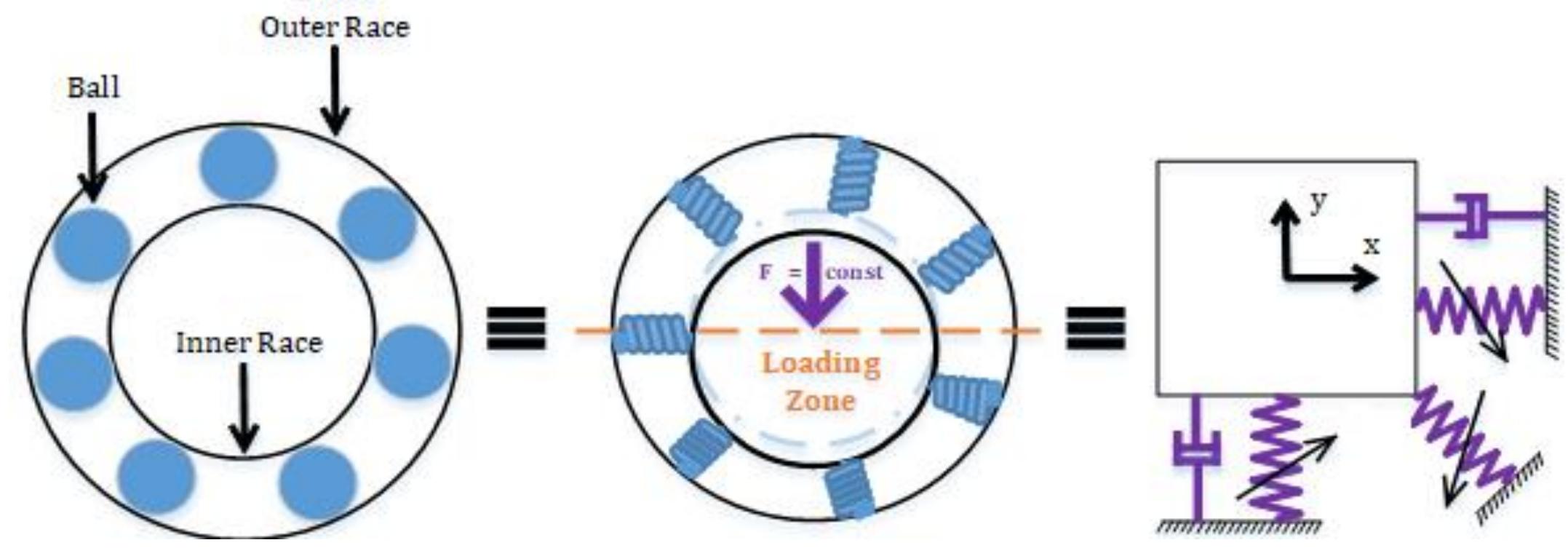
Abstract

The topic of this research is the vibration in rotors due to parametric excitation. The parametric excitation is known as an internally generated excitation that can lead to instability in rotating machines. A rolling bearing can be considered for the shaft a support with variable stiffness due to the change of the number of the balls in the loading zone during the shaft rotation.

In the past one year the attempt was to determine the stability and instability areas in the space of the parameters and the boundary lines of these areas, the so called Transition Curves. The Transition Curves can be obtained through a cost-intensive time-marching exploration of the input parameter space while in this study methods such as the harmonic balance methods, alternative to the time -marching, are applied to provide a complete picture of the transition curves.

Modeling & Methodology

The mathematical model of the bearing system generating parametric excitation is as below:



The equations of motion in the two directions x and y are written as:

$$\frac{d^2x}{dt^2} + \zeta \frac{dx}{dt} + [\delta - \epsilon_1 \sin 2\tau]x + [\epsilon_2 \cos 2\tau]y = 0 \quad (1)$$

$$\frac{d^2y}{dt^2} + \zeta \frac{dy}{dt} + [\delta + \epsilon_1 \cos 2\tau]y + [\epsilon_2 \sin 2\tau]x = 0 \quad (2)$$

Floquet Theory

The concept of theory is based on the Monodromy matrix which is built by fundamental solutions. Computing the eigenvalues of this matrix in the interval of $[0, T]$, the following conditions may occur:

$\lambda_i \in \mathbb{R}$	$ \lambda_i = 1$	$\lambda_i \in \mathbb{Z}$
$ \lambda_i < 1 \rightarrow$	$\lambda_i = 1 \rightarrow$	$ \lambda_i > 1 \rightarrow$
Stable behavior	T periodic solutions	Stable behavior
$ \lambda_i > 1 \rightarrow$	$\lambda_i = -1 \rightarrow$	$ \lambda_i < 1 \rightarrow$
Unstable behavior	$2T$ periodic solutions	Unstable behavior

Multiple Scales Method (MSM)

Applying this method the new time scales and time derivative are:

$$T_0 = \tau \quad (4) \quad \frac{d}{d\tau} = D_0 + \epsilon D_1 + \dots \quad (6)$$

$$T_1 = \epsilon \tau \quad (5) \quad \frac{d^2}{d\tau^2} = D_0^2 + 2\epsilon D_0 D_1 + \dots \quad (7)$$

Substituting Eq. (4) to Eq. (7) in Eq. (1) and Eq. (2), it would be found out that at $\delta = 1$ there exist small - divisor generators. This frequency is known as resonance frequency. To obtain the Transition Curves emanated from this frequency, δ would be defined as:

$$\delta = 1 + \epsilon \delta_1 + O(\epsilon^2) \quad (8)$$

Replacing Eq. (8) in the perturbation form of equations of motion and neglecting the secular term generators would result to the following:

$$\begin{cases} \frac{\partial A_1}{\partial T_1} \\ \frac{\partial B_1}{\partial T_1} \\ \frac{\partial A_2}{\partial T_1} \\ \frac{\partial B_2}{\partial T_1} \end{cases} = \begin{cases} -\hat{\zeta} - \frac{\hat{\epsilon}_1}{2} & \delta_1 & 0 & -\frac{\hat{\epsilon}_2}{2} \\ -\delta_1 & -\hat{\zeta} - \frac{\hat{\epsilon}_1}{2} & -\frac{\hat{\epsilon}_2}{2} & 0 \\ 0 & \frac{\hat{\epsilon}_2}{2} & -\delta_1 - \frac{\hat{\epsilon}_1}{2} & -\hat{\zeta} \\ \frac{\hat{\epsilon}_2}{2} & 0 & -\hat{\zeta} & \delta_1 - \frac{\hat{\epsilon}_1}{2} \end{cases} \begin{cases} A_1 \\ B_1 \\ A_2 \\ B_2 \end{cases} \quad (9)$$

[A]

Having periodic motion requires:

$$\lambda_A = 0 \rightarrow \delta_1 = \pm \sqrt{\frac{\hat{\epsilon}_1^2}{4} + \frac{\hat{\epsilon}_2^2}{4} - \hat{\zeta}^2} \quad (10)$$

Harmonic Balance Method (HBM)

According to Floquet theory when $\epsilon_2 = 0$, the responses along the TCs are periodic with T or $2T$ where $T = \pi$ is the period of the excitation. The solutions are assumed as:

$$\begin{aligned} x(t) &= a_0 + \sum_{n=1}^N (a_n \cos n\omega_s \tau + b_n \sin n\omega_s \tau) \\ y(t) &= c_0 + \sum_{n=1}^N (c_n \cos n\omega_s \tau + d_n \sin n\omega_s \tau) \end{aligned} \quad (11)$$

II & 2II Responses

Substituting the corresponding responses using Eq. (11) balancing the terms would result in the following:

$$[B_\pi] \{x_\pi\} = 0 \rightarrow |B_\pi| = 0 \quad (12)$$

$$[B_{2\pi}] \{x_{2\pi}\} = 0 \rightarrow |B_{2\pi}| = 0. \quad (13)$$

- When $\epsilon_2 \neq 0$ In this situation the dominant frequencies concerned with the solutions are ω_1 and ω_2 where:

$$\omega_2 = \omega_{ex} - \omega_1 \quad (14)$$

Considering the responses having this frequency and implementing HBM the following would be obtained:

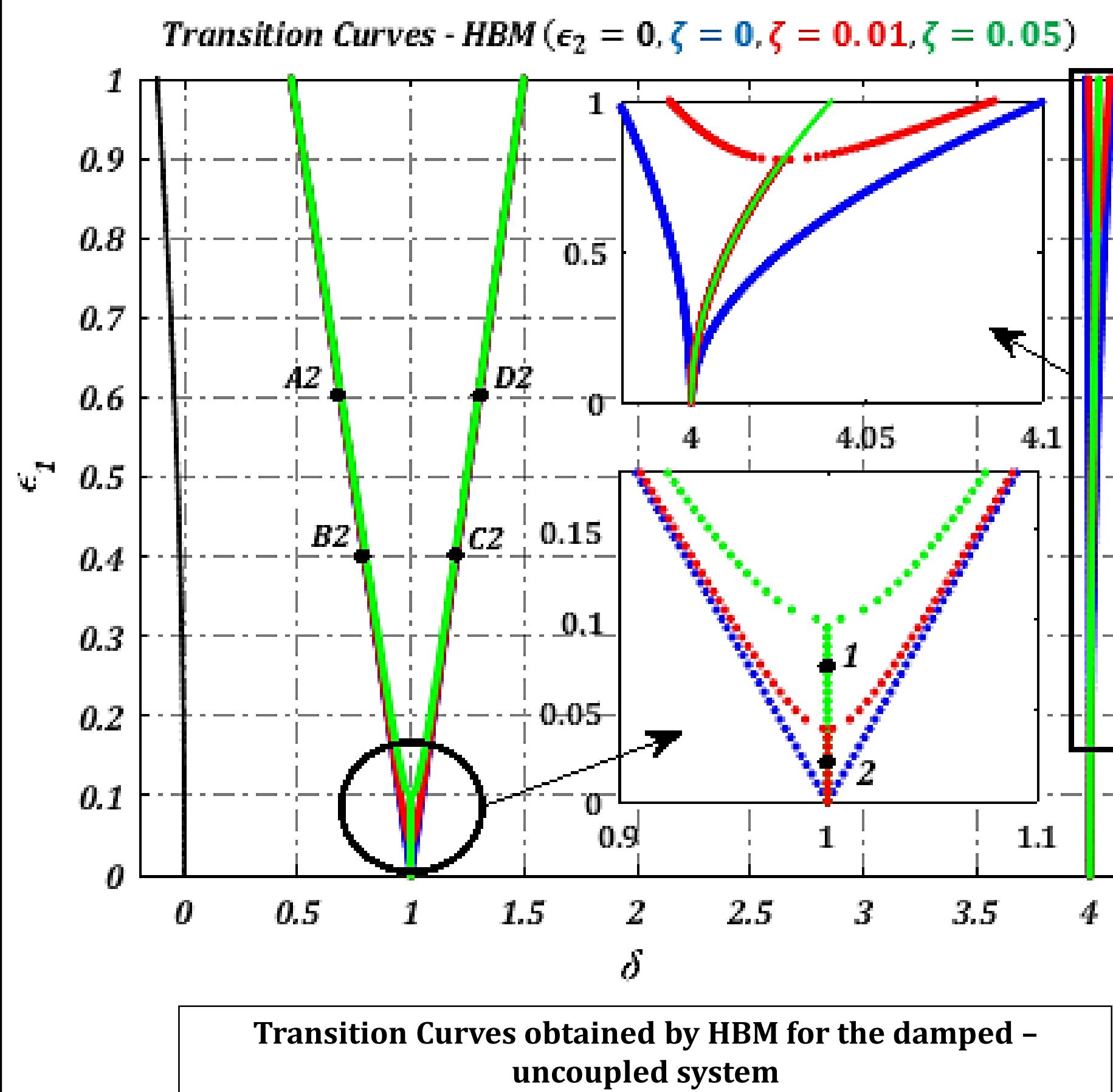
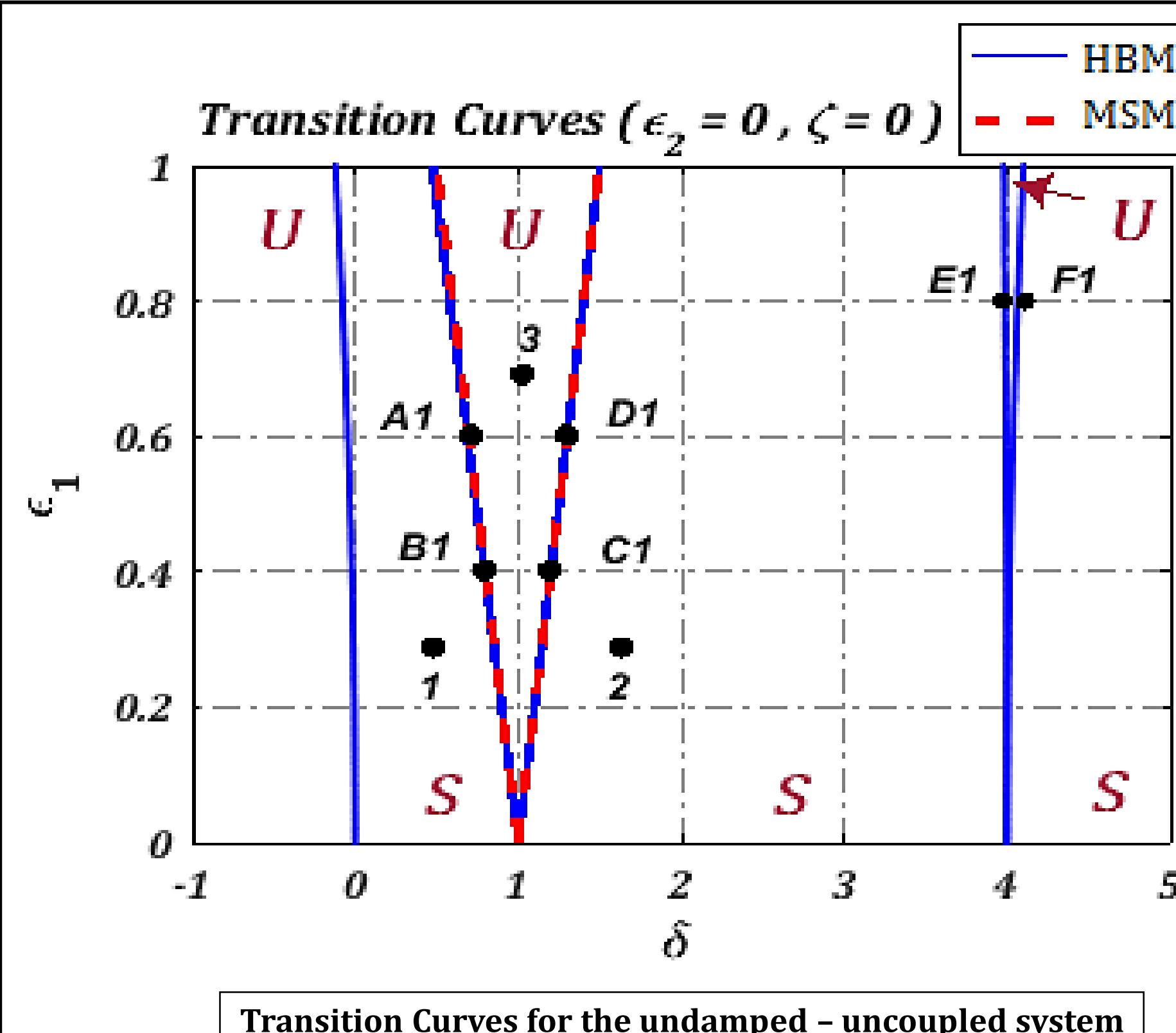
$$[B] \{x\} = 0 \rightarrow |B| = 0 \quad (15)$$

Sine the unknowns are δ , ω_1 and ω_2 , hence another equations in addition to Eq. (14) and Eq. (15) is required.

To this end the one of the minors of $[B]$ would be equal to zero:

$$|B|_m = 0 \quad (16)$$

Results



Training activities

Hard Skill Courses

- Fundamentals of fluid film lubrication: models and applications (12 hours)
- Waves in Periodic Structures and Elastic Metamaterials (20 hours)
- Numerical Modeling and simulation (50 hours)

Soft Skill Course

- Communication I (5 hours)
- Public speaking I (5 hours)
- Writing Scientific Papers in English (15 hours)
- Italian language I level

External Training Activities

- GTE Samara Summer School 2019 (Lectures + Practicals - 30 hours)

Conclusion

- For the cases uncoupled - undamped and uncoupled - damped bearing system the results of MSM and HBM are not only in line with each other but also are compatible with Floquet Theory.
- In the presence of the damping, the stability region increases
- In the case of coupled system, the TC get separated at the point of resonance frequency ($\delta = 1$) where the unstable regions expand.
- For the sake of HBM and MSM comparison, it is much more convenient to apply HBM specifically to calculate higher resonance frequencies and also when the number of the dominant harmonics are more than one (Coupled - Damped bearing system)